Practice Quiz 1 Math 202

Name: Solutions. Section: ____________

You have [redacted] minutes to complete the quiz. Please **show all work**, and then **circle your answer.**

1. Fill in the blanks, to complete the statement of Theorem 2:

   Theorem 2: The reduced echelon form of a linear system has three possible cases

   (a) The system has zero solutions if it contains a row $[0 \dots 0 | \blacksquare]$

   (b) The system has exactly one solutions if it is consistent and has a pivot in every coeff. column.

   (c) The system has ∞-many solutions if it is consistent and if some coeff column does NOT contain a pivot

2. For each of the cases above, write down an augmented matrix with the corresponding number of solutions.

   (a) zero
   $$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 4 \end{array}\right]$$
   requires $0 \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3 = 4$ but $0 \neq 4$

   (b) unique
   $$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

   (c) ∞-many
   $$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$
   (free variable in column 2)

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3. Write down the formal definition of Span$\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$.

the set of $\vec{b}$ s.t. $\vec{b} = c_1 \cdot \vec{v}_1 + c_2 \cdot \vec{v}_2 + c_3 \cdot \vec{v}_3$
for some c_1, c_2, c_3 in $\mathbb{R}$

4. What is the graphical meaning of Span$\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$?

it is the set of vectors that can be gotten by scaling & adding $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$

5. Fill in the blanks to state Theorem 4 in terms of pivots.

Theorem 4: The columns of an $m \times n$ matrix A span $\mathbb{R}^m$

if and only if there is a pivot in every Row of A

6. Write down a 3×3 matrix A whose columns span $\mathbb{R}^3$.

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

7. Write down a 3×3 matrix A whose columns do not span $\mathbb{R}^3$.

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

8. Can you write down a 2×3 matrix A whose columns span $\mathbb{R}^2$? Justify your answer.

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Yes.
the matrix to the left has a pivot in every Row
$\Rightarrow$ columns span $\mathbb{R}^2$

9. Can you write down a 3×2 matrix A whose columns span $\mathbb{R}^3$? Justify your answer.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

No.
you cannot have 3 pivot rows if you only have two columns.
$\Rightarrow$ no such matrix exist by Theorem 4.

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10. Write down the formal definition of Linear **Dependence** of vectors $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$.

$\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent

If there are c_1, c_2, c_3 in $\mathbb{R}$ NOT all 0
so that

$$c_1 \cdot \vec{v}_1 + c_2 \cdot \vec{v}_2 + c_3 \cdot \vec{v}_3 = \vec{0}$$

11. Give an example of a non-trivial dependence relation between $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3$.
Use this dependence relation to explain the graphical meaning of "Linear **Dependence**".

$$2\vec{v}_1 + 6\vec{v}_2 - \vec{v}_3 = \vec{0}$$

solving for $\vec{v}_3 = 2\vec{v}_1 + 6\vec{v}_2$
shows that $\vec{v}_3$ is in Span$\{\vec{v}_1, \vec{v}_2\}$.

12. What is the graphical meaning of the set $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$ being Linearly **Independent**?

If there is NO nontrivial dependence relation
Then NO $\vec{v}_i$ is in the span of the other remaining vectors.

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13. Suppose that $x_1\vec{\mathbf{v}}_1 + x_2\vec{\mathbf{v}}_2 + x_3\vec{\mathbf{v}}_3 = \vec{\mathbf{0}}$ has a unique solution.
Is $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$ Linearly **Independent**?

yes. $x_1 = x_2 = x_3 = 0$ is <u>always</u> a solution

<u>So</u> the only dependence relation is the trivial one.

<u>So</u> the vectors are linearly independent

14. Suppose that $x_1\vec{\mathbf{v}}_1 + x_2\vec{\mathbf{v}}_2 + x_3\vec{\mathbf{v}}_3 = \vec{\mathbf{0}}$ has infinitely many solutions.
Is $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$ Linearly **Independent**?

No. ∞-many solutions

$\Rightarrow$ has some nontrivial solution

$\Rightarrow$ there is a nontrivial dependence relation

15. Does $x_1\vec{\mathbf{v}}_1 + x_2\vec{\mathbf{v}}_2 + x_3\vec{\mathbf{v}}_3 = \vec{\mathbf{0}}$ always have a solution? Why or why not?

yes. $0 \cdot \vec{v}_1 + 0 \cdot \vec{v}_2 + 0 \cdot \vec{v}_3$

$= \vec{0} + \vec{0} + \vec{0}$

$= \vec{0}$

So $x_1 = x_2 = x_3 = 0$ is always a solution.